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THE MYSTERY OF TIME Maya Astronomy and Concept of Time

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THE MYSTERY OF TIME — MAYA ASTRONOMY AND CONCEPT OF TIME

JOHAN AARNES

ABSTRACT. As no other people in history the Maya sought to unite imaginations of time, space and gods in what we may call an astronomic-religious system. To understand the basic order of the Universe was an obsession which drove them through centuries. The highest goal for humans was to harmonize with this reality. To this end they made countless astronomical observations and computations which only can be improved in our time. The leading question in this article is: "How were they able to do it?"

We know a great deal about what they actually accomplished. They calculated the length of the year, the periods of the Moon and the planets and made tables of eclipses in a distant future. But surprisingly little is known about *how* they made their observations, *how* they calculated, and not least which mathematical tools they possessed. These are the questions we are going to pursue here, and indicate some answers.

1. INTRODUCTION

When I was a child I lived in a large house on a hillside near Oslo. This was during the war and there was a mandatory blackout on. Today it may be difficult to imagine a night in winter, close to a large city without any light to be seen. It was dark, I can tell. But the stars were there, clearer and more radiant than we can see them today, with the Milky Way as a magical ribbon in the sky, and a source of great wonder. It was the same sky the Mayas saw, some 1500 years ago. And they watched it, constantly. For them it also gave rise to another mystery: time.

It seems that in some profound way the two are linked: the mystery of the heavens and the mystery of time. Today it has a mathematical expression through the theory

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of relativity. But perhaps the Maya were the first to glimpse this connection. They were intensely preoccupied with time, as a concept and as a central element in their mythology. What is time, actually? Is there an absolute time? The Mayas *lived under time*, in the sense that they regarded time as divine and therefore deciding for their destiny and earthly life. Time was eternally flowing, without beginning or end. In their minds it was therefore linear, but they also looked at it as cyclic, recurrent and weighed down by events of the past. Occurrences did not happen only once, but repeated themselves in a pattern which they by astronomical observations made great efforts to understand. They lived in a universe where time, space and human destiny were linked together in a very complicated astronomical-religious system. The most prominent Maya scholar of the last century, Eric J. Thompson, expressed it in this manner:

No other people in history has taken such an absorbing interest in time as did the Maya, and no other culture has ever developed a philosophy embracing such an unusual subject. [6]

The most important god of the Maya was the Sun, who was the ruler of time and space. But also the Moon and the planets were deities, with different roles and influences. It was therefore of great importance to understand their movements in the sky. Of particular interest were conjunctions, as for example when Jupiter and Saturn were at approximately the same place in the sky. Such a meeting between gods was an event of cosmic dimension. It was thought of as a decisive moment and had to be calculated with great accuracy. This did not only concern future conjunctions, but also those that had already occurred. They meant that in order to understand the present and to predict the future, it was necessary to understand the movements of the gods in the past. This was a belief that permeated the society at all levels and influenced every day of the people. However, the religious beliefs of the Maya is not our concern here, but it may serve as a backdrop for our actual subject, the astronomical achievements of the Maya.

The principal time-measurer of the Maya was the Sun. It gave them what they called *kinh*, which in one word embraces day, sun and time. They observed regular events like solstice, equinox and not least, as we shall see, zenith. This gave them the year. But the moon also played a crucial role. This was another timekeeper which in fact is the key to the Mayas' complicated computation system. Without instruments of any kind, what is called "naked eye astronomy", they computed the length of the year, periods of the moon and the planets with a precision that surpassed Ptolemy, and they made tables of future eclipses. How did they do it? What mathematical tools did they possess? How did they perform their observations? These are the questions we are going to pursue.

2. The Maya Civilization

The Maya people lived, and still do, in an area consisting of the Yucatan peninsula in south-east Mexico and in Guatemala, Honduras and Belize. In time this civilization spans almost two thousand years and is without comparison the most accomplished and long-lasting culture in pre-Columbian time on the American continent. Other famous civilizations, as the Incas and the Aztecs, came much later and lasted only a few hundred years before the Spanish conquest put an end to them.

Mayan	mbe	r system
Sy	mbo	
0		
Rero	4	5
Ex	amp	1.4
		7
		11
-		20
:		21
<u> </u>		584

FIGURE 1. The number system. (By the author.)

2.1. Writing and numbers. The Maya enter the stage some 4500 years ago. Approximately 500 BC begins what is called the "formative period" of their civilization. During this period two decisive things happened. One of them was that they developed script. It was based on glyphs which could express syllables, morphemes and symbols. A complete writing system is characterized by that one may write what one wants and that it can be read and understood by the person reading it. History knows at most four, perhaps only two, civilizations which have accomplished this independently. The last, and guaranteed independent of others, was the Maya civilization, around 5–300 BC. The interpretation of the Maya script has not been an easy task, and it is only during the last 20–30 years that one may read most texts reliably [1]. The other thing that took place was the invention of an effective number system. The Mayas employed a system which is essentially vigesimal, meaning that it has base twenty, as opposed to our base ten system. It is a position system as ours, and it also has a symbol for zero, a shell, which is used both as a placeholder and as a symbol for nothing. The use of a shell as a symbol for zero makes one speculate if this invention occurred close to the sea, perhaps to facilitate the counting of a day's catch of fish [7].

2.2. Codexes and stelas. The main obstacle to an understanding of the Mayan way of thinking is the lack of written sources. There is a person in this story we can't ignore. Diego de Landa was a young priest when he arrived in Yucatan in the middle of the 16th century. He became deeply shocked over the religious ceremonies of the Maya, which he regarded as idolatry. He was elevated to bishop and became powerful, and he was clearly in possession of zealousness, holy wrath and administrative talent. He was so effective that when he ordered all their writings to be burned, only a few escaped the flames. Among these is what today is called the *Dresden Codex* the most important. It contains tables of astronomical observations of the Sun, the Moon and the planets, and lists of eclipses in a distant future. We do not know exactly what happened, if he regretted his actions or if he had other reasons, but in any case it is primarily by his efforts we can read Maya script today. By means of a Maya interpreter he constructed a first version of a kind of phonetic alphabet. He wrote a book: *Relación de las cosas de Yucatán*, which treated Maya script, customs, number system, calendar and not



FIGURE 2. Dresden Codex. Venus Tables (AD 700). From http://www.famsi.org/mayawriting/codices/dresden.html.



FIGURE 3. Stela in Coba (AD 780). (Photo by the author)

least, astronomy. It disappeared, however, and remained unknown until it reappeared in Madrid in the middle of the 19th century and gave the impetus to modern Maya research.

Fortunately, there are other sources. In addition to the codexes, of which only a few exist, there are inscriptions on walls and ceramics, and a great number of so called *stelas*. These are monuments in stone, with inscriptions. Most of them were erected in the *Classical period* of the Maya culture, 250–900 AD. We shall see that these stelas hold the key to the unlocking of the secret of the astronomical computations of the Mayas. Much of the honor for this work goes to a single person.



FIGURE 4. The temple in Chichen Itzá (AD 900). (Photo by the author)

2.3. The heroic work of Teeple. John E. Teeple was an American engineer, with a doctorate in chemistry, who among other things received a prize for the invention of a new method for the extraction of potash. He had a friend, Sylvanus G. Morley, who was one of the leading researchers in the decipherment of Maya texts. Morley encouraged Teeple to engage in the interpretation of inscriptions on stelas which gave dates and astronomical data, which they at that time, the mid 1920s, had been unable to figure out. Teeple met this challenge with great energy and inventiveness. The task was formidable. Teeple must have traversed an area larger than Norway, with bad roads, suffocating heat and an almost impenetrable jungle to locate and study virtually every stela there was to find. The work took 6 years and resulted in a long article: *Maya astronomy*, which appeared in the Carnegie institution series on American archeology in 1930 [9]. Even if this is a long time ago, and in spite of the great activity in Maya research today, Teeple's article stands. His results are quoted frequently, and I have not seen that he has been contradicted at any point concerning his interpretations and the computations he undertook. In the preface to the article he writes that

several pages relating to the tropical year may be unintelligible except to one already familiar with the Maya inscriptions and their subject matter. Others may study this section step by step and do the computations themselves.

This is what the author of the present article has done, with considerable effort it must be said, but hopefully so that it is intelligible also to the reader.

For a long time the general opinion was that the stelas mainly contained astronomical data. But as the decipherment of the script gained speed one now sees that they also contain information regarding wars, birth and death of kings, celebration of special events, as for example an astronomical conference in the year 687 AD. Suddenly this civilization has a history, like other nations [1, 3, 4].

2.4. Myths. In addition to the codexes and the stelas there is a third source, whose importance was long overlooked. The Mayas of today are still carriers of traditions and insights which may give a direct access to the past. But much of this knowledge is embodied in myths that are difficult to interpret.

One of the gods of the Maya was called Quetzalcoatl. He is probably identical with Kukulkan, who has his temple in Chichen Itzá. There is a special myth associated with this god.

They said that Quetzalcoatl died when the star became visible, and henceforward they called him the lord of dawn. They said that when he died he was invisible for four days; they say that he wandered in the underworld, and for four more days he was bone (dead). Not until eight days were past did the great star appear. They said that Quetzalcoatl then ascended the throne as god [7].

The physical phenomenon that corresponds to the Quetzalcoatl myth is the 8-day disappearance of the planet Venus in front of the sun. All the planets were gods, whereas phenomena as meteors and comets were dismissed as *the gods smoking cigars* [4]. Venus played a unique role in the minds of the Maya. They called it *Noh Ek*, which means the great star. The Mexican friar Toribia Motolinia writes that

Next to the sun they adored and made more sacrifices to this star than to any other celestial or terrestrial creature,... they knew on what day it would appear again in the east after it had lost itself or disappeared in the west...; they counted the days by this star and yielded reverence and offered sacrifices to it [7].

2.5. Venus. Let us take a few moments to consider how Venus moves across the sky. In its orbit around the sun it is inside us, between Mercury and the Earth. It completes one circuit in 225 days, while the Earth uses a little more than 365 days. It therefore changes position constantly, as seen from the Earth. When it is closest to us, it lies on a straight line between the Earth and the Sun. It is then invisible to us and will reappear only after some days. This is called the *heliacal* rise of the planet, the first time it is visible again in the east sky, for a fleeting moment before sunrise. Because of the advantage of having the inner track, Venus now runs away from us. The angular distance to the sun, as seen from the Earth, increases, and it may be seen for a longer time each morning. Its brightness increases and reaches a maximum after approximately 60 days, with a luminance surpassing all other stars or planets. But Venus is like a dog on a leash, it never moves more than 45 degrees away from the sun. After reaching its maximal angular distance it moves longer and longer away from us, while steadily losing in brightness. It then becomes invisible for about 7 weeks as it passes behind the sun. When it reappears it is as an evening star, to begin with close to the sun and almost invisible. Later, it comes closer to us and gains in luminance and again reaches its maximal brilliance as a shining evening star. Then it again comes close to the sun and fades in brilliance and finally becomes invisible as it passes in front of the sun another time until it rises anew. The full cycle takes about 584 days to complete and is called the *synodic period* of Venus.

For the post-classic Maya, the planet Venus was a male god: the light-skinned Quetzalcoatl-Kukulcan, who wore a beard. Possibly based on a real person who was exiled in the tenth century from Tula, the Toltec capital of Central Mexico, he was said to have journeyed east to Yucatan to found a new empire. He predicted his own return to Tula, and by a quirk of history, Cortez invasion of Mexico in 1519 coincided precisely with the heliacal rise of Venus. Montezuma, King of the Aztecs laid down his arms in anticipation that the god had returned to claim his kingdom.

In earlier times, the heliacal rise of Venus held particular importance. It was closely connected with the coming of the rainy season and the planting of their most important crop, maize. It was also considered an especially fortunate time to go to war. It appears that the Mayan cities were constantly waging wars against each other. The object was not primarily looting and conquest, even if that certainly happened. The main purpose was to procure slaves. In the classical period the Mayas always had large building projects going on. In a dense tropical jungle, under blistering heat, this must have been an unhuman struggle. For a wise ruler, the sensible thing to do was to employ foreign labor, so that his own citizens remained content. The slaves were also utilized as players in football games, where the losing team in general was sacrificed.

The Maya counted a Venus period to be the time from one heliacal rise to the next, and quite accurately put it to be 584 days. However, after observations over many years they realized that the average period actually was a little shorter. They introduced corrections, meticulously written down in the Dresden Codex on pages 46 to 50. These corrections amounted to approximately one day every twenty years. They were incorporated in tables local priests could consult, in order to know when ceremonies should take place. From the Codex one may deduce that in the span of 301 periods of Venus of 584 days, one should subtract a total of 24 days. This is so accurate that it gives an error of less than two hours in 500 years, or more precisely: 24 minutes per century!

2.6. Astronomy and architecture. It had long been thought that several buildings have orientations related to positions of Venus at the horizon. One may think this to be strange, since heliacal risings of Venus follow the sun closely, and the point at the horizon where the sun sets or rises varies a great deal during the year. The explanation may be found in an article, Venus and the Maya [7], by Anthony Aveni. He is professor in astronomy at Colgate University in USA, and is a pioneer in what now is called archaeoastronomy. Together with a mathematician colleague and a graduate student, in the 1980s he undertook a detailed exploration of Venus alignments in three cities, Copán, Uxmal and Chichen Itzá, and made astounding discoveries. Aveni's theory was that the Maya were particularly interested in Venus' extreme southerly or northerly rise. The southernmost heliacal rise of Venus must occur a few minutes before sunrise around winter solstice. One should think this to be a rare event. But because of a chance astronomical fact: 5 synodic periods of Venus is very close to 8 years, this phenomenon will repeat in cycles of 8 years. The shift in position at the horizon is so little that it will take more than a hundred years before it is noticeable. There is an example from Uxmal which is quite spectacular.

They found that the line from the center of the middle doorway in the *Governors Palace* to a man-made mound 6 kilometers away, deviated from a perpendicular on the facade by less than 8 minutes of arc. More importantly, when Venus made its first appearance in its greatest southerly rise, probably on January 11, 751 AD, it would stand exactly over the top of this mound, as seen from the middle doorway. Another example of Venus alignments may be found in the so-called *Caracol* in Chichen Itzá. Aveni found several lines of sight here, corresponding to Venus positions and



FIGURE 5. The Observatory. (Photo by the author)

observations of equinoxes. The Venus calendar in the Dresden Codex may well be based on observations from this tower.

3. The Maya Calendar

The calendar of the Maya is a mathematical piece of art in itself. It consists of two parts, or perhaps we should call it two counters, which run alongside without stopping. One part, which is called *Tzolkin*, has a cycle of 260 days until it starts over again. The other part of the calendar simply is a 365 day year which is divided in 18 months of 20 days each, plus a short month of 5 days which is called Wayeb. This period of 365 days is commonly called *Haab*, even if it appears that this not a Maya name. Tzolkin was a religious calendar, which gave dates for religious events and ceremonies, whereas the Haab year was a profane calendar which gave dates for planting and harvesting.

3.1. **The Tzolkin.** In Tzolkin there are 20 day names which always repeat in the same order, Imix is the first and Ahau is the last. A *date* is given by the day's name and a number from 1 to 13 which run in sequence, and starts over again continuously. The least common multiple of 13 and 20 is 260, so there will be no repetition of combinations of name and number until the full cycle of 260 days is complete.

3.2. The Haab year. The 18 ordinary months of the Haab year also have names and a glyph representing the god that rules over each month. The 5 days of the month Wayeb were considered unlucky. These days were not protected by any god, so the powers of the underground had free reign. One had to take precautions, not undertake any serious work or wash ones hair. It was also said that anyone born on one of these days would always remain poor and have a miserable life (De Landa). A day's position in the month is numbered from 0 to 19, except Wayeb, which runs from 0 to 4. A date will be given, for example as 5 Pop, like we say March 5. By combining the Tzolkin date and the date in the Haab year we get a *complete date*, as for example 4 Ahau 8 Cumhu. A little analysis shows that no complete date will repeat until 18980 days have passed. This amounts to 52 Haab years, and is commonly called a calendar round.

The 365-day Haab year is a little shorter than the solar year, or tropical year, as the astronomers call it. We compensate by adding an extra day every four years. The

Mayas knew quite well about this difference, but it did not seem to concern them. And it must be admitted that such a rigid counting system has its advantages, as with adjoining cogwheels running for ever without missing a notch. However, to a modern person, two things appear very strange. Firstly, the calendar is cyclic, repeating itself in 52 year cycles endlessly. Secondly, it has no year counter. Now, if we think about it, the first part is perhaps not so strange after all. The Gregorian calendar also repeats itself when it comes to dates and names of the days of the week. The calendar of an ordinary year of 365 days repeats with intervals of 5 or 8 years, and the calendar of a leap year repeats after 28 years. In this cycle of 28 years all 7 possible calendars will occur. For an ordinary year 3 times, and for a leap year just once. But the second fact, that there is no number which tells how far time has progressed in the full cycle of 52 years, must have left most people with a feeling of floating in time, without anchorage. The 52-year cycle always starts with the date 1 Imix 0 Pop and ends with 13 Ahau 4 Wayeb, but in between we only have the dates, with no year indication. So how did a person know his age, for instance. Of course, if a person's date of birth occurred once more, he would know that he had lived a full cycle. Probably not many people lived to be 52 years at that time. But if it actually happened to a person, it gave cause to great celebrations. However, the Maya had still another time-keeping system.

3.3. The Long Count. To fasten events in the past and in the future they had a quite different method, they invented a system which is equally simple as it is ingenious. They counted days. It is described in de Landa's book, in Spanish it was called *La Cuenta Larga*, which translates into *The Long Count*. Even if the Maya considered time as infinite in both directions, they had a mythological point of reference. Most Maya researchers agree that this date is August 11, -3113 (or 3114 BC by the back-dated Gregorian calendar). According to Maya mythology this was when the last creation occurred, when man was created from maize. It is also inside the time period where one thinks that the people of Meso-America changed from being solely hunters and gatherers, to doing agriculture.

The Long count system was employed throughout the Mayan world on a consistent basis. This is indeed a fortunate fact, for stelas in various locations carry inscriptions which give dates for astronomical events. These recordings are our main source to understanding Maya astronomy and computations.

The Maya name for day was *kinh*. Twenty kinh made a month, which was called a *uinal*. 18 uinals make a *tun*, which consequently consists of 360 days. A tun is therefore a shortened 365-day year, we throw away the 5 bad days of Wayeb. We adopt Teeple's system and take tun as the basic unit in the long count. Twenty tuns make a *katun*, and twenty katuns make a *baktun*, and so on, but the higher tuns are rarely used and need not concern us here.

A date in this system may for instance be: 3 baktuns, 8 katuns, 14 tuns, 7 uinals and 5 kinh, counted from the mythical creation date. Teeple writes this as 3.8.14-7-5. The first three numbers give the year, meaning the number of 360-day periods, while 7 gives the month and 5 the day. This is quite similar to our way of writing a date, for instance 2016-10-28.

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Name	Days	Solar years (approx.)
Kinh	1	
Uinal	20	
Tun	360	1
Katun	7200	20
Baktun	144000	394

TABLE	1.	The	Long	Count



FIGURE 6. Stela with creation date. From Wikipedia "Maya Calendar".

How did the Maya write such dates? Let us look at a very interesting inscription on Stela C in Quirigua which gives the date of point Zero in their chronology. The date we read is 13.0.0-0-0, 4 Ahau 8 Cumhu, see Figure 6. Here we must add that the Maya always counted *elapsed time*, as opposed to our system counting *current time*. The first day of a new year was therefore indicated by 0 Pop, while we write January 1. We do not know why the long count dating was reset after the completion of Baktun 13, such that the next date would read 0.0.0-0-1. We may note that here is also the calendar round date included, which was quite common practice. This implies that all calendar round dates are locked for all future with respect to the long count.

3.4. The end of the world. For fun, let us perform a little computation. 13 baktuns is $13 \times 144000 = 1872000$ days. According to the Gregorian calendar a year is 365.2425



FIGURE 7. Supplementary series. From [9, p.42].

days. Division gives 5125.3619 years. Starting from -3113 and adding 5125 we get 2012. Next, 0.3619 year equals 132 days. Counting 132 days from August 11, we arrive at 21 December 2012. As we may recall, this was the date when the world was supposed to go under. Maybe it actually did, just that we haven't realized it yet. But for the Maya this date was simply the transition to a new cycle, which according to myth, shall bring peace and harmony among the peoples.

4. The Moon

4.1. **Supplementary series.** We are not quite finished with the time-keeping of the Maya. As for most other civilizations the moon played an important part in their daily life and as a time-keeper. By recordings which cover many hundred years they were able to compute the the length of a moon period with extreme accuracy. However, as we shall see later, the real importance in this result lies in that they used it as a vehicle for determining the length of a solar year. To understand how they thought, it is necessary to look at the inscriptions more closely. The Maya counted moons modulo 6, that is: moon periods from new to new in cycles from 1 to 6, and then started over again. 6 moons make a lunar half year, and two periods of 6 moons is a lunar year. The *age* of the moon was the number of days after a new moon and will vary from 0 to 29 or 30.

There is a group of six or seven glyphs, called the supplementary series, which have to do with the moon. In general they follow immediately after the long count date. It turns out that the glyphs C, D and E are of particular importance (see Figure 7). Teeple is given the honor of the decipherment of these glyphs, which we are going to take a closer look at. The supplementary series attached to a date 1.18.5-3-6 on the Temple of the Sun in Palenque records glyph C with the number 4 and glyph E with the number 6. The supplementary series attached to the date 1.18.5-4-0 found in another temple, also in Palenque, records glyph C with the number 5 and the glyph D with the number 10. We may write this as 1.18.5-3-6; 4C, 6E and 1.18.5-4-0; 5C, 10D.

These two dates are only 14 days apart. What plausible explanation will convert 4C, 6E to 5C, 10D fourteen days later? Teeple asks. Teeple's assumption was that 4C means 4th Moon and 5C means 5th Moon. He guesses that 10D is the age of the moon, so the inscription means 10 days in the 5th Moon. What about 6E? He guesses that E stands for 20, such that 6E becomes day 26 of the 4th moon. The assumption is therefore that D with numbers from 0 to 19 is used for moons of age less than 20 days, E with numbers from 0 to 10 is used for moons of age 20 days or older. This interpretation was tested by Teeple on a great number of inscriptions, and he found that it was correct. This was a decisive breakthrough. The crucial point here is that the long count now was brought in contact with verifiable astronomical events.

The end of a katun was often marked by the erection of a stela with inscriptions of astronomical data. In his article Teeple gives a long table of moon numbers and ages at katun endings from different monuments. The table lists the age of the moon at this date, as it was observed, and how it was computed by Teeple on the basis of the actual synodic period of the moon, 29.53059 days. There may be a difference of one or two days, something which is quite natural for real observations. The registered age of the moon may be checked on more inscriptions than all other astronomical data put together, says Teeple. There is no wonder that these data did not always agree. In general, there was consensus about the age of the moon, if there was a new moon in Palenque, it was also a new moon in Copan. But there could be differences when it came to which moon, if it was number 4 or 5 in the cycle, for instance.

4.2. The synodic period of the moon. What do we get out of this? The table spans more than 400 years of observations of the moon. It is clear that there are enough data to give a rather good computation of the synodic period of the moon, the average time between two consecutive new moons. But the Maya did not think this way. They did not work with fractions, only with whole numbers, and they wanted to find a formula for the exact number of days which gave a definite number of moons.

Below we list four inscriptions of dates with moon positions which may be found on different monuments in Palenque. They are all inscribed in or shortly after AD 678. The inscription on Stela 1 records an actual observation, whereas the other three represent computed Moon positions, meticulously calculated by the astronomers in Palenque. These dates are exceptional in the sense that they specify events several thousand years in the past, long before the time when writing and a number system was in place. We may perhaps regard this as a kind of show off, to demonstrate how clever they were. Note that the inscription on the Temple of the Cross indicates a date even earlier than the date of creation! But it is more to it than that. For Teeple this was a kind of Holy Grail. The computed positions reveal that the same formula must have been used in all three cases.

Monument	Date	Gregorian year	Moon number	Moon age
Stela 1	9.12.6-5-8	678	5	19
Temple of the Cross	12.19.13-4-0	3120 BC	2	5
Temple of the Sun	1.18.5-3-6	2359 BC	4	26
Tpl. foliated cross	1.18.5-4-0	2359 BC	5	10

TABLE 2. Moon positions

The formula they had found at Palenque was 81 moons equal 2392 days, and this is the formula they must have employed to calculate the moon positions in the distant past listed above. For example, the time interval between the date on the Temple of the Cross and the date on Stela 1 amounts to 1387108 days. $(1387108/2392) \times 81 = 46971, 4667$ moons. $46971 = 3 \mod 6, 0,4667 \mod \approx 14$ days. Stela 1 is 5/19 (moon number/moon age), subtracting 3/14 we get 2/5 as inscribed at Temple of the Cross. The other time intervals also give a perfect match with the inscribed moon positions, indeed confirming that the same formula: 81 moons equal 2392 days, was used in all three cases.

If we do the division it yields a moon period of 29.53086 days, while the correct value is 29.53059 days. However, there was no complete agreement about this formula. There was a serious controversy, in particular between Palenque and Copan, clearly expressed on many stelas. After the script became more fully understood, we may read this today as polemic literature engraved in stone. We will say more about this controversy a little later, and why this was a question of the greatest importance.

4.3. The enigma of the dates. Let us pause and consider the following questions: Why were these particular dates in the past chosen? and Why are there no dates telling when the inscriptions were done? The fact that the inscription date not was given was contrary to normal practice, but is perhaps the easiest question to answer. As we intend to show, the inscription date is irrelevant for the computation they undertook. What mattered was the time span involved. We intend to undertake a little "mathematical archeology".

To ask why these particular dates in the past were chosen is not just idle curiosity. By pursuing the question, I think we may learn a little more about how the Maya reasoned, and also about how they actually did their computations. It is clear that these dates can not have any relation to celestial or historic events. There could be no records going so far back in time. Any coincidence with a heliacal rise of Venus, for instance, would be accidental. We have to look elsewhere for a clue. A reasonable assumption could be that the inscriber would choose a date in the past such that the time span from this date to the present date, would yield the same moon numbers. If we look at the date in The Temple of the Cross, 12.19.13-4-0; 2,5 the most striking aspect is that it predates the mythical creation date 13.0.0-0-0, and also specifies the moon positions. This is indeed a daring leap in imagination. It is a like trying to estimate the state of the Universe some time before The Big Bang.

As for the actual computation the astronomers undertook we know next to nothing, all traces of these are lost. But we do know the results ensuing from these computations. However, mathematics is a rigorous discipline. which tells us that to get from A to B you have to perform certain operations. For instance, if a Maya King wanted to build a pyramid of a certain size, he would have to perform various arithmetical operations, including multiplication and division, to calculate the number of blocks of limestone, the number of slaves and the time needed to get the job done within his lifetime. The Maya probably did not have an algorithm either for multiplication or division, but that does not mean that they were unable to perform these operations. Multiplication could be done by repeated addition or by referring to tables, which they surely must have constructed. Division could be performed by inverse multiplication, that is, by multiplying the divisor sufficiently many times and then end up with a remainder in most cases.

Now, if we look at the date on Stela 1, we find that it is 1,384,668 days after their point Zero, 13.0.0-0-0. Division by 2392 yields 578 (plus remainder) = number of periods of 81 moons, an arithmetic operation we must assume that they were able to perform, or perhaps they already knew from previously computed tables. This number is close to

580, which, as we shall see, is a very good number when it comes to Maya computation. The astronomers of Palenque must have decided, one day not long after the erection of Stela 1, that they wanted to compute the moon positions on a date prior to date Zero. Assuming that 580 periods of 2392 days was what they used, the day they must have chosen as a starting point turns out to be 9.12.7-0-0. This is just 252 days later than the date on Stela 1, so they knew that going 580 periods of 81 moons back in time, they would end up with a date prior to date Zero. Now, why is 580 such a good number?

In the first place $580 \times 81 = 46980 =$ number of moons in this period. This number is divisible by 6, which it must be if we are going to have the same moon numbers. Secondly, 580 is the *smallest number of integral periods of 81 moons, divisible by 6,* which will produce a date earlier than the Zero-date. Finally, the time span involved is $580 \times 2392 = 1,387,360$ days, is exactly equal to 5336 Tzolkins.

Starting backwards in time from 9.12.7-0-0, how would they calculate to arrive at this date 12.19.6-5-8 inscribed on the Temple of the Cross? For readers with a stomach for arithmetic calculations, we sketch one possible approach in the Appendix. We only mention here, that odd as it may seem, a key element in the calculation is the fact that $580 = 5 \times 116$.

If we turn to the date on the Temple of the Sun: 1.18.5-3-6, the time difference to the date on Stela 1 is 1,109,202 days. Dividing this by 2392 we get 463 (plus remainder) periods of 81 moons. This number is close to 464, a very fortunate — or more likely, an intended fact. In this case we have $464 = 4 \times 116$. We again refer to the Appendix for the actual calculation.

It is the hope of the author that the reader will agree with him that the conclusion is that the dates on the Temple of the Cross and the Temple on the Sun were not chosen in advance, but are results of mathematical convenience, under the constraint that the number of moons involved must be divisible by 6. In the case of The Temple of the Cross we have the extra property that the time span is an integral number of Tzolkins. This is certainly not accidental, such coincidence will happen only once every 810 moons, or about each 65 years. It must be a deliberate act by the Maya computers, and demonstrates quite clearly their computational skill and their preference for synchronized cycles.

4.4. The controversy between Copan and Palenque. In Copan they had found another formula, they operated with 149 moons = 12-4-0 = 4400 days. If we do the division this gives a synodic period of 29.53020 days, a very good approximation indeed, although not quite as good as the one they had found in Palenque. However, it was not the formulas per se which was at the heart of the conflict, it was their consequences. In Copan there is an inscription with the date 9.12.8-3-9 which records fifth moon with age 22 days. This is exactly 5 days earlier than the date 9.12.8-3-14 in Palenque, so judged from Palenque the moon data should be 4 and 21, as opposed to 5 and 22 in the inscription. This a serious discrepancy. A difference of one day can be put down to observation, but different moon numbers are another matter. We must bear in mind that the moon was divine. It was mainly a godess, with great influence on fertility, rain and human fortune. The gender could change to a male god for a full moon, perhaps even more powerful [4]. Since also numbers were divine, the changing attributes of the moon seem to be closely connected to the two key numbers, moon number and moon age. In short, this was not a question of astronomy, it was a question of theology. And as we know, this is when the discussion really heats up. We may just think of the dispute in Islam about whether it was the father in law or the nephew of Mohammad who was the true successor of the prophet (shia vs. sunni), or the correct interpretation of Trinity in the Christian Church.

The discussion went on for some 40 years. To to put an end to it, in the year 687 AD was held what we must call an Astronomy conference in Copan, with delegates from all the large cities. Here some agreement must have been reached, for in the next century or so there are no discrepancies between inscriptions of moon numbers. More importantly, it was the astronomers of Copan who won the battle about the formula for the moon period. After this defeat, Palenque disappeared, literally. The loss of prestige must have been so severe that they gave up and the city was abandoned. Or one may suspect that foul play was involved, that the delegates from Palenque simply were eliminated there and then. Paradoxically, posterity has given Palenque a kind of revenge. Copan's formula gives a synodic period which is 34 seconds to short, while Palenque had a period which was 23 seconds too long.

We close this section by a somewhat curious observation. If we take the average of 81 and 149, and the average of 2392 and 4400 and combine the two averages we get 115 moons equal 3396 days. Strangely enough, this formula does not appear anywhere. It corresponds to a moon period of 29.53043 days which is just 14 seconds short.

5. The solar year

Let us proceed to something else of even greater importance. How long is a year? As we know, a year is the time it takes for the Earth to complete one orbit around the Sun. More precisely, a tropical year is the time between two passages through vernal equinox. We know that the Earth moves in an elliptic orbit with the Sun in one focal point. The Maya did not know it, and no other people on Earth at that time either. The Maya concept of the Earth was a large flat disk, surrounded by water on all sides. The Sun rose and moved in its arc over the sky, to set and disappear in the underworld, where it had to defeat the powers of darkness, to return and rise again the next day. The Sun's position at the equinoxes gives a possibility to determine the length of a year. For the Maya a year was the time it took for the Sun to return to the same position after completing a cycle, for instance by observing the day when the rays from the Sun make a straight line, from sunrise to sunset, which happens at equinox. However, this method has its drawbacks. It requires free sight to the horizon and that the horizon is not obscured by clouds. There are other problems as well, which we don't have to go into. The conclusion is that the Maya certainly knew that these were special days, but it is doubtful that they used the equinoxes as the basis for computation of the solar year. They had another method which did not suffer from the same problems. In the area of the Maya, which entirely lies in the tropical zone, the Sun will reach zenith two times every year, symmetrically located in time with respect to summer solstice. In many Maya cities there are *zenith tubes*, vertical shafts into the ground with a cave for observation underneath. But in Chichen Itzá $(20^\circ 40' N, 88^\circ 34' V)$ they had something even better.

5.1. Zenith observations. A *cenote* is a natural pit, or sinkhole resulting from the collapse of limestone bedrock that exposes groundwater underneath. In the Yucatán Peninsula, in addition to being the principal source of water, they were sometimes used for sacrificial purposes. But they also had a third kind of employment.

Let us try to compute the length of the year as I believe the Maya may have done it, by means of zenith observations, exemplified by the Holtun-cenote in Chichen Itzá. This cenote is circular and almost completely closed, there is only a small square hole in the roof permitting sunlight to enter. From the ceiling it is approximately 30 meters down to the water surface. Steps are cut in the walls permitting access to the water. We must assume that the Mayas in one way or another rigged up some kind of platform above the water, and that they did some markings on it. When the Sun nears zenith, the hole in the roof will be pictured as a sun patch moving across the platform. We may imagine that the zenith position is localized by means of a plumb line from the roof and indicated as a kind of target on the platform. However, in most years the Sun will not reach zenith *exactly*, it will miss the target by a small margin. It is the fact that it misses, and by how much it misses, which gave the Maya observers the key to the computation. What they saw the days before a zenith day was the path of the sun patch passing on one side of the target, but getting nearer each day. The maximal altitude of the Sun at midday now changes by approximately 0.2 degrees per day, which results in a translation of the path of the sun patch by about 10 cm, which is clearly observable. With respect to accuracy of the observations, the great height of the cenote is important.

To make it simple, let us assume that the sun patch actually hit bulls eye in what we may call year zero. Now let 365 days pass. What we see then, and what the Maya observers saw, was that the center of the sun patch passes 2.5 cm to the left of the target. This is not very much, but certainly observable. If we wait another 365 days, we see a new translation to the left by 2.5 cm. We may continue like this for 4 periods of 365 days and obtain a total displacement of 10 cm. But 10 cm is the displacement of the path from one day to the next. Therefore, if we wait one more day, 366 days in all, we get an almost perfect hit. This gave the Maya their first approximation. As we know, they did not work with fractions. Their way of expressing the result was: After 4 Haab years and one day the Sun is again in the same position, almost. There was a small error, which became clearer after observations over many years. Their next approximation was that after 104 Haab years, they had to add 25 days. This result corresponds to a solar year of 365.2404 days, which is a better approximation than Ptolemy's.

For the Maya this was still not good enough. Their quest for accuracy was nothing less than fantastic. The problem however, is that it takes a very long time to obtain a precise answer.

Well, how did they proceed? Teeple discovered that the Maya astronomers had a completely different method than observations over hundreds of years. The Maya were



FIGURE 8. Glyphs on Stela A in Copan. From [9, p. 71].

obsessively interested in the attunement of cycles. Consequently, the thing to do was to synchronize the periods of the Moon with a certain number of solar years.

5.2. The stela in Copan. As we already know, they had very good control on the moon. The solution to the problem was found on what is called Stela A in Copan. There is a long inscription, but only three dates.

9.14.19-8-0, 12 Ahau 18 Cumhu

9.15.0-0-0, 4 Ahau 13 Yax

9.14.19-5-0, 4 Ahau 18 Muan

The second date indicates that this was a monument marking the end of Katun 15 (AD 731). The last date is of special interest. 19-5-0 = 6940 days, which is as close to 19 tropical years one may come with whole numbers. 19 years is the Metonic cycle, which was known in Babylonia several hundred years before the Greek astronomer Meton gave his name to it in the year 433 BC. 19 years is very close to 235 moons, so it seems possible that the astronomers of Copan had discovered this connection. By studying the glyphs on Stela A closer, Teeple found this: (See Figure 8). Here a and b are the glyphs for Imix and Ahau, respectively, the first and the last name of days in the month, but which also signifies beginning and end. e and f are glyphs of moon and sun adjoined. This indicates quite convincingly that by the date 9.14.19-5-0, the Sun and the Moon were at the same position with respect to each other as they were at the end of Katun 14, so that 235 moons equal exactly 19 solar years. And how did they know when 19 solar years had passed? The cenote instrument we just described provided the answer. It is a sun-clock. It does not give the hours of the day, but with good accuracy it tells the year, by studying zenith passages. And we know that they knew how to count moons. Here are the formulas of the astronomers of Copan.

19 years = 235 moons

149 moons = 4400 days

Put together, we see that these two formulas contain enough information to compute the length of a tropical year.

 $19 \text{ years} = 235 \text{ moons} = (235/149) \times 4400 \text{ days} = 6939.597 \text{ days}$

Division by 19 yields:

1 tropical year = 365.2420 days

Here it is appropriate to remark that neither of the formulas employed in the computation are exact. 149 moons is a small fraction of a day more than 4400 days, and 19 tropical years is a small fraction of a moon less than 235 moons. It turns out that these two errors cancel each other almost exactly in the computation, so that the result is almost exactly right. As a consequence we may think that the astronomers of Copan are given more credit for accuracy than they deserve. However, it is not unlikely that they were aware of the discrepancies, even the fact that the errors cancel each other out. We must keep in mind that through zenith observations they had an independent source of information against which they could check their results. Here is a table of the length of a tropical year, calculated at various times.

Correct value today	$365.2422~\mathrm{days}$
Duration around AD 600	$365.2423~\mathrm{days}$
Gregorian year	$365.2425~\mathrm{days}$
Ptolemy, around AD 200	365.2467 days
Maya (Copan)	$365.2420~\mathrm{days}$

It is likely that the astronomers of Copan were very pleased with themselves after this accomplishment. Now they had full control over the periods of the moon and the solar year, and how their cycles fit together. There exists what Teeple calls a group portrait of The Academy of Science of Copan from AD 731. It consists of portraits on a stone wall which he believed indicated the celebration of an event. Unfortunately, more recent research have shown that these figures are not astronomers, but the succession of kings that ruled in Copan [2].

5.3. The ultimate calculation. Now, having these formulas, they wanted to use them for all their worth, in a rather incredible way. The idea was to calculate all the way back to the day of creation, the point Zero of their chronology. Their goal was to find out how far the solar year had progressed in relation to the 365-day Haab year, since point Zero.

As we already have mentioned, we do not know how the Maya did their calculations. All traces are lost, only the results remain, written in stone. However, it is clear that in order to obtain the results they achieved, they needed all four arithmetical operations. In the Appendix we give a brief sketch of how they could have proceeded to solve the problem. The conclusion is that they found that on the date 9.15.0-0-0, 3846 Haab years of 365 days plus 210 days had passed, while 3844 tropical years plus 10 days had gone by since point Zero.

Comparison then tells us that the tropical year has advanced two whole 365-day years + 200 days = 930 days in all, in relation to the Haab year. How did they record this extraordinary result? The number 930 is nowhere to be found. We must go back to the inscriptions on Stela A in Copan to find the answer. The first inscription on this stela gives the date 9.14.8-0-0. This date is exactly 200 days before the second date 9.15.0-0-0.

A number of later inscriptions on other stelas confirm this computation [9]. This was the intellectual culmination of the Maya classical period. They achieved astronomical results fully on level with and which in fact surpassed many of earlier civilizations.

6. Reflections and Conclusion

The epoch we have heard about here, was the flowering of the Maya civilization. It produced spectacular buildings, grand cities, impressive pieces of art and great intellectual discoveries. The period lasted for about 600 years, until the last stelas were erected

around 900 AD. It was followed by stagnation, internal struggles and a gradual decline. The large cities were abandoned, one after another, and in the course of time, almost completely forgotten. What happened?

Inherent in the Maya culture there are some paradoxical elements. They constructed and built impressive buildings, but they had almost no technology. They had a circular calendar, but they did not have the wheel, not even a potters wheel. They performed complicated calculations, but they had virtually no geometry. And not least, they had developed an astronomy explaining the movements of the heaven, but they never asked the question about why.

And we must admit that if the bodies that moved across the sky were gods, and the gods were moody and capricious, then this an impossible question to ask. So, they were trapped in their own imagination of the world, as so many cultures before them had been. There was no room for a Thales, an Aristotle or an Archimedes. If one of them had shown up, he most certainly had been put to a quick end.

However, this does not explain the collapse of the Maya culture. Several theories have been advanced, as for example: It had exhausted its energy, they were conquered by the Toltecs who had a different culture, prolonged periods of drought, et cetera. All of this may have contributed, but recent research points at another cause, which is both banal and tragic: famine [11]. The Maya had only three nutritious plants; maize, squash and beans, where maize was the most important. They had only two domestic animals; turkey and duck. Game could add to the menu, but was probably mainly for the higher levels of society. In addition they had cocoa and wild honey to sweeten it. Their form of agriculture was not sustainable when the cities became to large, and the relation between people producing food and a non-productive upper class became unbalanced. The soil in large parts of Yacatán is poor, after a few years of harvesting it was burned off and laid idle for some years. Even more dramatic were the consequences of deforestation. They needed wood for firewood, but most of the forest was cut down in connection with the building projects. The Maya liked to cover their walls with stucco, which was an excellent base for decorations. However, to produce lime they had to burn large quantities of limestone. This required large quantities of wood. The subsequent deforestation led to erosion and landslides which could partly bury cities and fields, as it happened in Copan, which was subsequently abandoned. It is modern archeology combined with microbiological studies of human remains which has led to the conclusion that the collapse of the Maya society was caused by a lack of nourishment.

After working with the discoveries and fate of the Maya for almost two years one is left with mixed feelings. There is fascination, respect and a great deal of sadness. It is tempting to ask if they could have learned from their mistakes and regained their footing, perhaps even gotten rid of their Spanish rulers. There is an episode which is quite telling. In the middle of the 17th century the Maya had become thoroughly tired of the Spanish rule. They managed to put their differences aside and mounted an army. They rebelled and were in fact on the verge of defeating the Spaniards when they suddenly stopped. Instead they went home to harvest the maize which was ripe.

7. Appendix

7.1. Calculation of two dates in the past. The calculation which follows is a construction by the author, and is of course just one of several possibilities. The task is to multiply the basic unit 2392 = 6-11-12 by 580. Now $580 = 5 \times 116$, so this may be done in two steps: first multiplying 6-11-12 by 116, and then multiplying the result by 5. Now 116 = 100 + 16 and $100 = 5 \times 20$. In a true vigesimal system multiplication by 20 is accomplished by moving each number one place to the left (or up, depending on the way the total number is written) and inserting a zero (a shell) at the place to the extreme right (or at the bottom). The Maya dating system is vigesimal, with the exception of the second place (the uinals) which only runs to 18. Some adjustment therefore had to be made, which the Maya certainly were capable to do. The result is $20 \times (6 - 11 - 12) = 6.12 - 16 - 0$, thus $100 \times (6.11.12) = 5 \times (6.12 - 16 - 0) = 1.13.4 - 8 - 0$. Next, we compute $16 \times (6 - 11 - 12)$. $16 = 2 \times 2 \times 2 \times 2$, so this may be done by repeated additions. Skipping the details, we get 5.6-5-12. Adding this to 1.13.4-8-0 we obtain $116 \times (6 - 11 - 12) = 1.18.10 - 13 - 12$. Finally, $580 \times (6 - 11 - 12) = 5 \times (1.18.10 - 13 - 12) = 9.12.13 - 14 - 0$. Subtracting this number from the assumed or fictitious date 9.12.7-0-0, we obtain 12.19.13-4-0 as expected.

Next, to compute back to the date 1.18.5-3-6, we first note that $464 \times 2392 = 1,109,888$. This gives a date 686 days after the date 9.12.6-5-8 on Stela 1. 686 = 1-16-6, adding this to 9.12.6-5-8 we get 9.12.8-3-14. This is our assumed date as basis for the computation. Now 464 = 580 - 116, so to compute back we may utilize the numbers calculated above. Hence the time difference corresponding to 464 periods of 81 moons equals 9.12.13 - 14 - 0 - 1.18.10 - 13 - 12 = 7.14.3 - 0 - 8. Subtracting this number from our new starting date 9.12.8-3-14 we get 1.18.5-3-6 as expected. Since $464 \times 81 = 37584$ is divisible by 6, 9.12.8-3-14 must have the same moon numbers as the date on the Temple of the Sun: 4 and 26.

7.2. Progress of the Solar year from point Zero. Our computation differs somewhat from Teeple's, and is hopefully somewhat easier to follow. The starting point is the date 9.15.0-0-0, the end of the 15th Katun. It places us 1,404,000 days after point Zero. It is likely that they had control on the number of 365-day years which had passed. Otherwise, they turned to their form of division: 1,404,000 divided by 365 is 3846 plus remainder 210, because $365 \times 3846 = 1,403,790$. Hence the end of the 15th Katun is 3846 Haab years of 365-days plus 210 days after the beginning of year Zero.

Now they needed to find out how many solar years which had passed. From the formulas it is clear that they had to convert days into moons, and then convert moons into solar years. We proceed by steps.

- Division of 1,404,000 by 4400 yields 319 with remainder 400. Thus, 319 periods of 149 moons = 47531 moons, plus 400 days have elapsed.
- Dividing 47531 by 235 we get 202 with remainder 61 moons. Multiplying 19 years by 202 yields 3838 tropical years.

We have left out 61 moons and 400 days and need to convert them into tropical years, and add the total to 3838. Now 61 moons equals a little more than 1801 days, we may write this as $1801 + \epsilon$ where ϵ represents a small fraction of a day.

• Adding the 400 days we have left out we get $400 + 1801 + \epsilon = 2201 + \epsilon$

Now 6 solar years equal a little more than 2191 days, we put 6 solar years $= 2191 + \epsilon$.

• Consequently the end of Katun 15 is 3838 + 6 = 3844 tropical years, plus $(2201 + \epsilon) - (2191 + \epsilon) = 10$ days after the creation.

References

- [1] Coe, Michael D. (1994) Breaking the Maya Code. London: Thames & Hudson.
- [2] Laughton, Timothy. (1998) Life, Myth and Art: The Maya. Duncan Baird Publ. Ltd.
- [3] Leon-Portilla, Miguel. (1988) Time and Reality in the Thought of the Maya. second ed., enlarged. University of Oklahoma Press, Norman.
- [4] Milbrath, Susan. (1999) Star Gods of the Maya. Austin, Texas. University of Texas Press.
- [5] Tedlock, Barbara. (1992) Time and the Highland Maya. Albuquerque, New Mexico. University of New Mexico Press.
- [6] Thompson, J. Eric S. (1974) Maya Astronomy. Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences. London, UK: The Royal Society. 276 (1257, The Place of Astronomy in the Ancient World): 83–98.
- [7] Aveni, Anthony F. (1979) Venus and the Maya. American Scientist, Vol. 67, 274–285.
- [8] Aveni, Anthony F. (2001) Skywatchers. (originally published as: Skywatchers of Ancient Mexico [1980], revised and updated ed.) Austin, Texas. University of Texas Press.
- [9] Teeple, John E. (1931) Maya Astronomy. Contributions to American Archaeology. Volume I (Pub. 403 ed.), 29–116.
- [10] Voss, Alexander (2006) Astronomy and Mathematics. In Nikolai Grube (ed.). Maya: Divine Kings of the Rain Forest. Cologne, Germany: Könemann, pp. 130–143.
- [11] Diamond, Jared (2005) Collapse. Viking, Penguin Group, New York.

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